2.6 Properties of Equality and Congruence

Properties of Equality and Congruence

Reflexive Property a = a (Or ≅)
Ex: AB = AB $\overline{AB} \cong \overline{AB}$
m∠1 = m∠1 ∠1 ≅ ∠1

Symmetric Property If a = b, then b = a. (Or ≅)
Ex: If AB = CD, then CD = AB.
If ∠1 ≅ ∠2, then ∠2 ≅ ∠1.

Transitive Property If a = b, and b = c, then a = c. (Or ≅)
Ex: If AB = CD, and CD = EF, then AB = EF.
If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

The Addition Property—If a = b and c = d, then a + c = b + d.
(Add an equal value to BOTH sides of the equation.)

Ex. If x - 8 = 9 then x = 17

Ex. If y - 4 = 2 then y = 6

The Subtraction Property—If a = b and c = d, then a - c = b - d.
(Subtract an equal value from BOTH sides of the equation.)

Ex. If x + 2 = 9 then x = 7

Ex. If AB = CD then AB - EF = CD - EF

The Multiplication Property—If a = b, then a•c = b•c.
(Multiply an equal value to BOTH sides of the equation.)

Ex. If $\frac{1}{2}x = 10$ then x = 20

Ex. If m∠1 = m∠2 then 2·m∠1 = 2·m∠2

The Division Property—If a = b and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

(Divide an equal value from BOTH sides of the equation.)

Ex.

If $2x = 40$

then $x = 20$.

Ex.

The Substitution Postulate—If a = b, then a can be substituted for b in any equation or inequality.

Ex.

If x + y = 13 and y = 5, then x + 5 = 13.

Ex.

If $m\angle 2 = 40$ and $m\angle 1 = m\angle 2$,

then $m\angle 1 = 40$

Fill in the reasons why for each step.

Examples:

1.

$2x - 3 = 13$ Given

$2x = 16$ Addition

$x = 8$ Division

2.

$4x = 8$ Given

$x = 2$ Division

3.

$\frac{1}{2}x + 2 = 11$ Given

$\frac{1}{2}x = 9$ Subtract

$x = 18$ Mult./Division

4.

$7 = 3x - 5$ Given

$12 = 3x$ Addition

$4 = x$ Division

$x = 4$ Symmetric

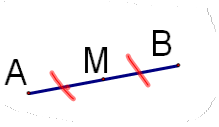
Justify each statement.

5. Reflexive ___ $m\angle 1 = m\angle 1$
6. Addition ___ If $m\angle 1 = m\angle 2$, then $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$
7. Multiplication ___ If $AB = CD$, then $2 \cdot AB = 2 \cdot CD$.
8. Symmetric ___ If $RS = XY$, then $XY = RS$
9. Transitive ___ If $m\angle A = m\angle B$, and $m\angle B = m\angle C$, then $m\angle A = m\angle C$
10. Division ___ If $2 \cdot m\angle 1 = 90$, then $m\angle 1 = 45$

Other things we already know:
Vertical ∠s are ≅; Def. of midpoint; Def. of angle bisector; Def. of supplementary angles; Def. of complementary angles; Segment addition postulate; Angle addition postulate; Congruent complements theorem; Congruent supplements theorem

Give the reason for the conclusions below.

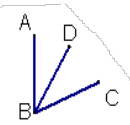
11. Given: M is the midpoint $\overline{AB}$
Conclusion: $AM = MB$
Reason: def of midpoint



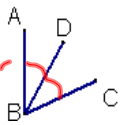
12. Given: ∠1 and ∠2 are complementary.
Conclusion: $m\angle 1 + m\angle 2 = 90$
Reason: def. of complementary

13. Given: diagram to the right
Conclusion: CA + AT =CT
Reason: Segment Add. Postulate

14. Given: diagram
Conclusion: m∠ABD + m∠DBC = m∠ABC
Reason: Angle Addition Postulate



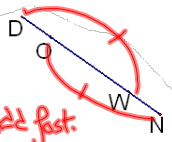
15. Given: $\overrightarrow{BD}$ bisects ∠ABC
Conclusion: m∠ABD = m∠DBC
Reason: def of Angle Bisector



16.

Ex. 7 Given: DW = ON
Prove: DO = WN

Statements	Reasons
1. DW = ON	1. Given
2. DW = DO + OW; ON = OW + WN	2. Segment Add Post.
3. DO + OW = OW + WN	3. Substitution
4. OW = OW	4. Reflexive
5. DO = WN	5. Subtraction



Reasons to choose from for #16:
Reflexive; Given; Segment Addition Postulate; Substitution; Subtraction